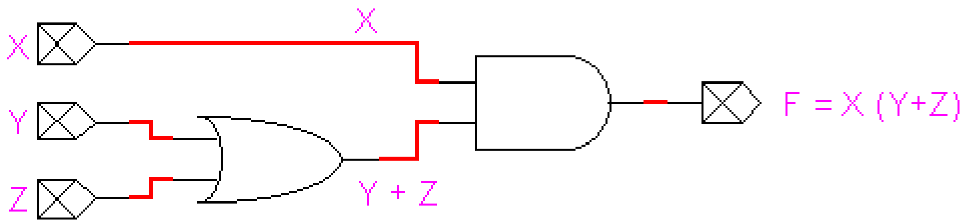
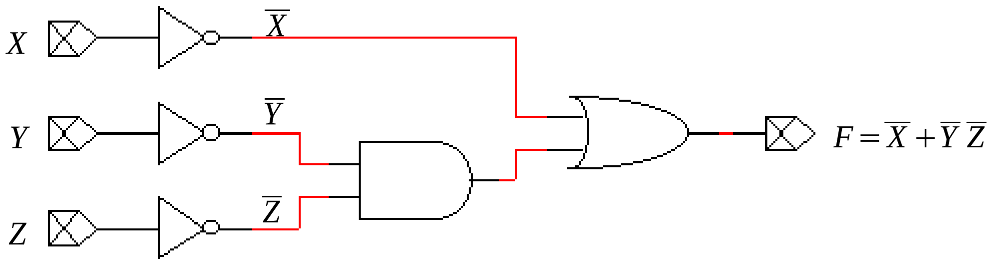
Esercizi svolti

1. Date le seguenti funzioni logiche ricavare le corrispondenti reti logiche realizzate con porte elementari AND, OR, NOT.

a) $F = X(Y + Z)$

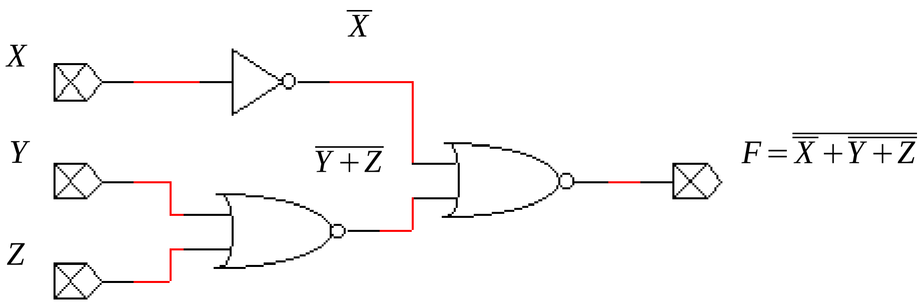


b) $F = \overline{X} + \overline{Y}\,\overline{Z}$

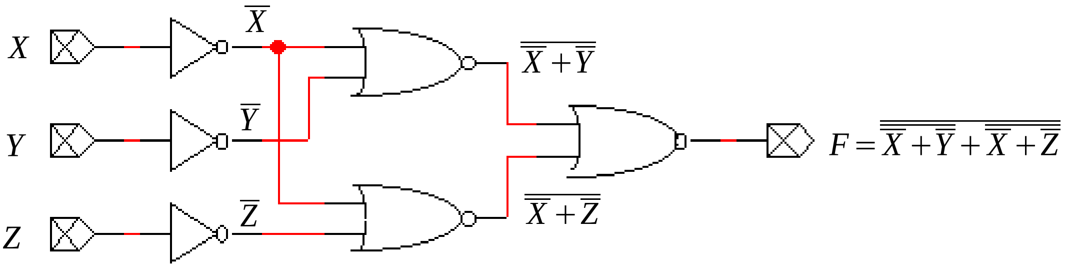


2. Date le seguenti funzioni logiche ricavare le corrispondenti reti logiche realizzate con porte elementari NOR e NOT.

a) $F = \overline{\overline{X} + (\overline{Y + Z})}$

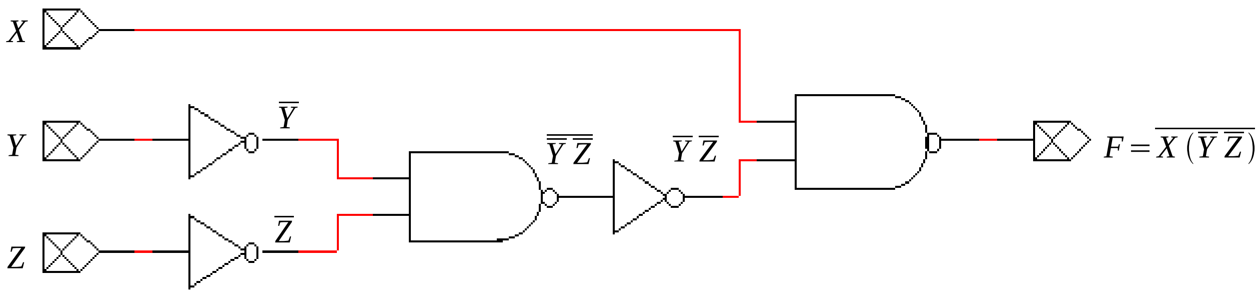


b) $F = \overline{\overline{\overline{X} + \overline{Y}} + \overline{\overline{X} + \overline{Z}}}$

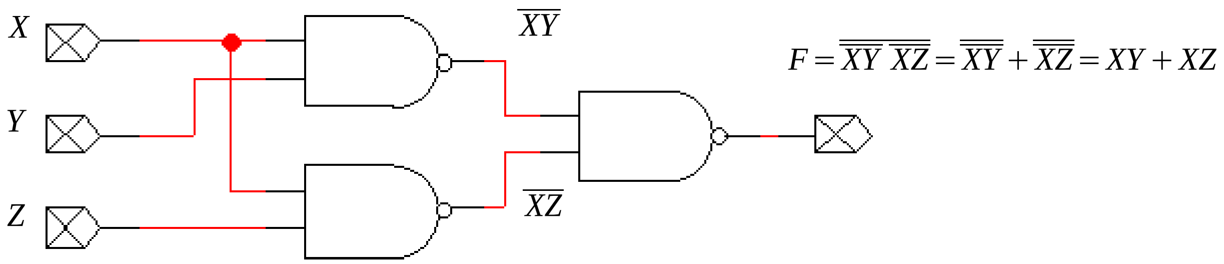


3. Date le seguenti funzioni logiche ricavare le corrispondenti reti logiche realizzate con porte elementari NAND e NOT.

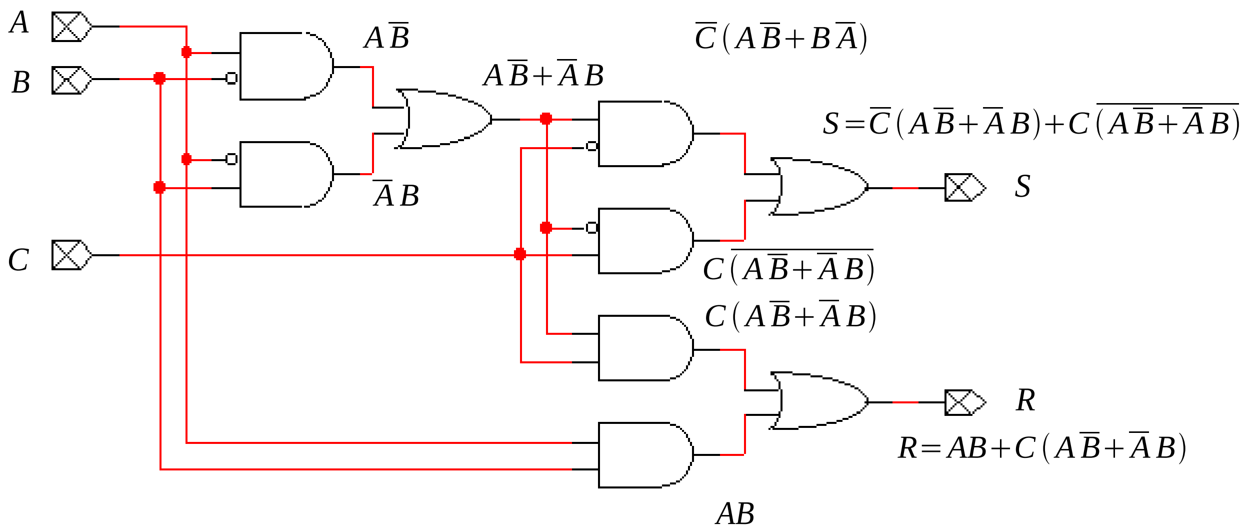
a) $F = \overline{X\,(\overline{Y}\,\overline{Z})}$ $\qquad F = \overline{X\,(\overline{Y}\,\overline{Z})} \Rightarrow F = \overline{X\,\overline{\overline{(\overline{Y}\,\overline{Z})}}}$



b) $F = XY + XZ$



4. Data la rete logica di figura ricavare le corrispondenti funzioni combinatorie in forma minima



$$S=\overline{C}(A\overline{B}+\overline{A}B)+C(\overline{A\overline{B}+\overline{A}B})$$
$$S=\overline{C}(A\oplus B)+C(\overline{A\oplus B})$$

$$\overline{A\overline{B}+\overline{A}B}=A\odot B \qquad A\overline{B}+\overline{A}B=A\oplus B$$

$$S=A\oplus B\oplus C$$

T9, Distributiva

$$R=AB+C(A\overline{B}+\overline{A}B)$$
$$R=AB+A\overline{B}C+\overline{A}BC$$

T8, Assorbimento: AB + ABC + ABC = AB ,

$$R=AB+ABC+ABC+A\overline{B}C+\overline{A}BC$$

T6, pr. Commutativa

$$R=AB+A\overline{B}C+ABC+\overline{A}BC+ABC$$

T10', combinazione

$$R=AB+AC+BC$$

5. Dimostrare per manipolazione algebrica la proprieta' del consenso T11': .

T11' : $XY+\overline{X}Z+YZ=XY+\overline{X}Z$

$XY+\overline{X}Z+YZ$ — T10' pr. della combinazione

$$XY+\overline{X}Z+YZ=XYZ+XY\overline{Z}+\overline{X}YZ+\overline{X}\,\overline{Y}Z+XYZ+\overline{X}YZ$$

XY $\qquad$ $\overline{X}Z$ $\qquad$ YZ

T6 pr. Commutativa

T3 idempotenza

$$XY+\overline{X}Z+YZ=XYZ+XYZ+\overline{X}YZ+\overline{X}YZ+\overline{X}\,\overline{Y}Z+XY\overline{Z}$$

T6 pr. Commutativa

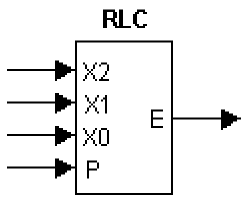
$$XY+\overline{X}Z+YZ=XYZ+\overline{X}YZ+\overline{X}\,\overline{Y}Z+XY\overline{Z}$$

T10' pr. della combinazione

$$XY+\overline{X}Z+YZ=XYZ+XY\overline{Z}+\overline{X}YZ+\overline{X}\,\overline{Y}Z$$

$$XY+\overline{X}Z+YZ=XY+\overline{X}Z$$

6. Dato un segnale a 4 bit (con ultimo bit di parità) indicare la tabella di verità di una rete logica che verifichi se la parità e' corretta e costruire la corrispondente mappa di Karnaugh.



(parita' pari)

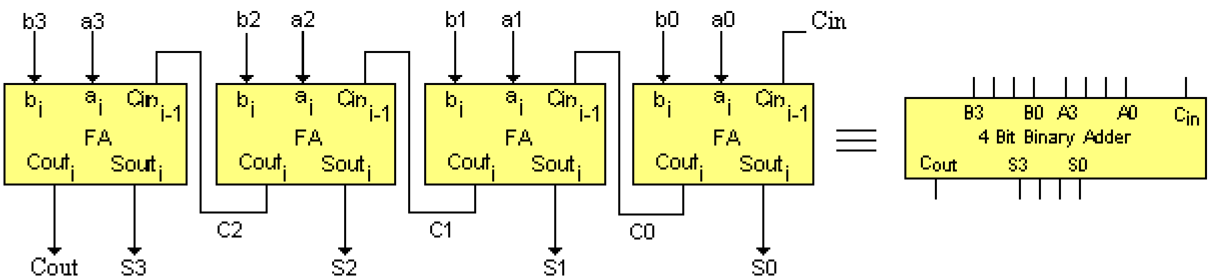
Tabella di verita'

P	X_2	X_1	X_0	E
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**1**
0	0	1	1	**0**
0	1	0	0	**1**
0	1	0	1	**0**
0	1	1	0	**0**
0	1	1	1	**1**
1	0	0	0	**1**
1	0	0	1	**0**
1	0	1	0	**0**
1	0	1	1	**1**
1	1	0	0	**0**
1	1	0	1	**1**
1	1	1	0	**1**
1	1	1	1	**0**

Mappa di Karnaugh

$P\,X_2$ \ E, X_1X_0	0 0	0 1	1 1	1 0
00	0	1	0	1
01	1	0	1	0
11	0	1	0	1
10	1	0	1	0

7. Full Adder: Ricavare la tabella di verità e le mappe di Karnaugh per una rete logica che effettua l'addizione ad un bit con riporto in ingresso ed in uscita.



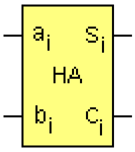
Cin
A = (a3, a2, a1, a0)
B = (b3, b2, b1, b0)

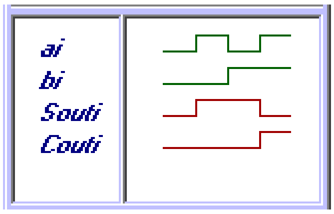
S = (Cout, s3, s2, s1, s0)

Descrizione:

La somma tra numeri binari e' effettuata da reti logiche denominate half adder e full adder. Tali reti operano su parole ad 1 bit. Interconnettendo tra loro piu' reti si ottengono circuiti logici in grado di operare con parole ad n bit.

Half adder (mezzo sommatore) ad un bit (effettua la somma tra due parole ad i bit)





Half Adder			
a_i	b_i	Si	C_i
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Si: a_i \ b_i	0	1
0	0	1
1	1	0

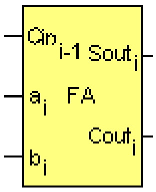
C_i: a_i \ b_i	0	1
0	0	0
1	0	1

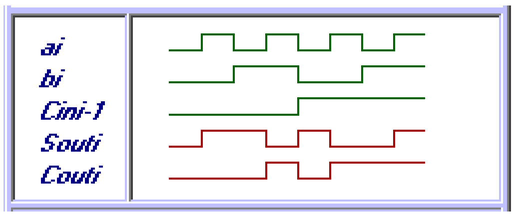
Blocco funzionale e tabella di verità'

Diagramma dei tempi e mappe di Karnaugh

Full adder

(sommatore completo) ad un bit (effettua la somma tra due parole ad un bit, tiene conto del riporto precedente)





Blocco funzionale e tabella di verità'

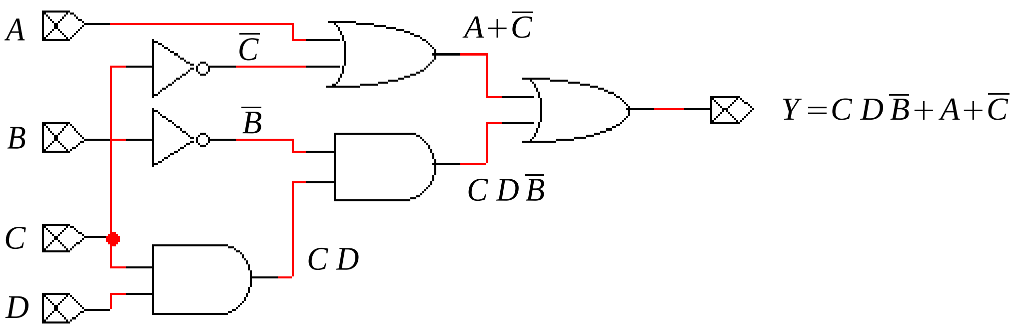
Full Adder				
a_i	b_i	Cin_{i-1}	$Sout_i$	$Cout_i$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$Sout_i$: a_i \ b_i Cin_{i-1}	0 0	0 1	1 1	1 0
0	0	1	0	1
1	1	0	1	0

$Cout_i$: a_i \ b_i Cin_{i-1}	0 0	0 1	1 1	1 0
0	0	0	1	0
1	0	1	1	1

Diagramma dei tempi e mappe di Karnaugh

8. Data la rete di figura ricavare la funzione logica in forma algebrica. Semplificare la funzione combinatoria.



$Y=C\,D\,\overline{B}+A+\overline{C}$ — T8, Assorbimento aggiungo $\overline{C}\,D\,\overline{B}$

$Y=C\,D\,\overline{B}+A+\overline{C}+\overline{C}\,D\,\overline{B}$ — T6, pr. Commutativa

$Y=A+\overline{C}+C\,D\,\overline{B}+\overline{C}\,D\,\overline{B}$ — T10' pr. della combinazione

$Y=A+\overline{C}+D\,\overline{B}$

Altro modo:

$Y=C\,D\,\overline{B}+A+\overline{C}$ — T6, pr. Commutativa

$Y=A+\overline{C}+C\,D\,\overline{B}$

$Y=A+\overline{C}+D\,\overline{B}$

Assorbimento 2:

T8" $X+\overline{X}\,Y=X+Y$

T8"' $X\,(\overline{X}+Y)=X\,Y$

D	C	B	A	$A+\overline{C}+D\,\overline{B}$	
0	0	0	0	1	$\overline{C}$
0	0	0	1	1	$A,\overline{C}$
0	0	1	0	1	$\overline{C}$
0	0	1	1	1	$A,\overline{C}$
0	1	0	0	0	
0	1	0	1	1	A
0	1	1	0	0	
0	1	1	1	1	A
1	0	0	0	1	$\overline{C},D\,\overline{B}$
1	0	0	1	1	$A,\overline{C},D\,\overline{B}$
1	0	1	0	1	$\overline{C}$
1	0	1	1	1	$A,\overline{C}$
1	1	0	0	1	$D\,\overline{B}$
1	1	0	1	1	A $D\,\overline{B}$
1	1	1	0	0	
1	1	1	1	1	A

9. Semplificare per manipolazione algebrica la seguente espressione booleana.

T9', pr. Distributiva

$$(A+B+\overline{C}+D)(A+\overline{B}+\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})$$

T3', pr. Idempotenza

T5', pr. Complementarieta'

$$(AA+A\overline{B}+A\overline{C}+AD+BA+B\overline{B}+B\overline{C}+BD+\overline{C}A+\overline{C}\,\overline{B}+\overline{C}\,\overline{C}+\overline{C}D+DA+D\overline{B}+D\overline{C}+DD)(A+\overline{B}+\overline{C}+\overline{D})$$

T8, pr. Assorbimento (A)

T8, pr. Assorbimento $(\overline{C})$

$$(A+A\overline{B}+A\overline{C}+AD+BA+B\overline{C}+BD+\overline{C}A+\overline{C}\,\overline{B}+\overline{C}+\overline{C}D+DA+D\overline{B}+D\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})$$

T8, pr. Assorbimento (D)

T9', pr. Distributiva

$$(A+\overline{C}+D)(A+\overline{B}+\overline{C}+\overline{D})$$

T3', pr. Idempotenza

T5', pr. Complementarieta'

$$AA+A\overline{B}+A\overline{C}+A\overline{D}+\overline{C}A+\overline{C}\,\overline{B}+\overline{C}\,\overline{C}+\overline{C}D+DA+D\overline{B}+D\overline{C}+D\overline{D}$$

T8, pr. Assorbimento (A)

$$A+A\overline{B}+A\overline{C}+A\overline{D}+\overline{C}A+\overline{C}\,\overline{B}+\overline{C}+\overline{C}D+DA+D\overline{B}+D\overline{C}$$

T8, pr. Assorbimento $(\overline{C})$

$$A+\overline{C}+D\overline{B}$$ Forma minima !!

10.Verificare che le seguenti espressioni risultano duali:

Principio di dualità

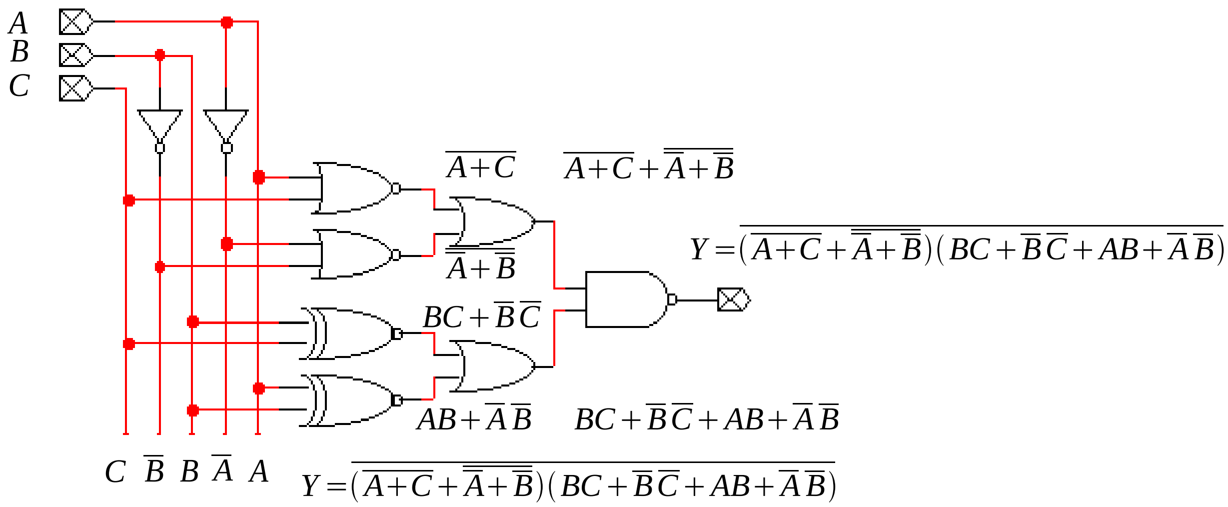
a) NOR e NAND
b) EXOR e EXNOR

NOR e NAND $Y=\overline{A+B} \Rightarrow Y'=\overline{AB}$

EXOR e EXNOR $Y=A\oplus B=\bar{A}B+A\bar{B} \Rightarrow Y'=(\bar{A}+B)(A+\bar{B})=\bar{A}A+\bar{A}\bar{B}+BA+B\bar{B}$

$Y'=\bar{A}\bar{B}+AB=A\odot B$

11.Ricavare la funzione logica in forma algebrica e semplificare applicando i teoremi dell'algebra booleana.



$$Y=(\overline{\overline{A+C}+\overline{\bar{A}+\bar{B}}})+(\overline{BC+\bar{B}\bar{C}+AB+\bar{A}\bar{B}})$$

$$Y=(\overline{\overline{A+C}})(\overline{\overline{\bar{A}+\bar{B}}})+(\overline{BC+\bar{B}\bar{C}})(\overline{AB+\bar{A}\bar{B}})$$

$$Y=(A+C)(\bar{A}+\bar{B})+(\overline{BC+\bar{B}\bar{C}})(\overline{AB+\bar{A}\bar{B}})$$

$$Y=(A+C)(\bar{A}+\bar{B})+\overline{BC}\,\overline{\bar{B}\bar{C}}\,\overline{AB}\,\overline{\bar{A}\bar{B}}$$

$$Y=(A+C)(\bar{A}+\bar{B})+(\bar{B}+\bar{C})(\bar{\bar{B}}+\bar{\bar{C}})(\bar{A}+\bar{B})(\bar{\bar{A}}+\bar{\bar{B}})$$

$$Y=A\bar{A}+A\bar{B}+C\bar{A}+C\bar{B}+(\bar{B}+\bar{C})(\bar{\bar{B}}+\bar{\bar{C}})(\bar{A}+\bar{B})(\bar{\bar{A}}+\bar{\bar{B}})$$

$$Y=A\overline{B}+C\,\overline{A}+C\,\overline{B}+(\overline{B}+\overline{C})(B+C)(\overline{A}+\overline{B})(A+B)$$

$$Y=A\overline{B}+C\,\overline{A}+C\,\overline{B}+(\overline{B}\,B+\overline{B}C+\overline{C}\,B+C\,\overline{C})(\overline{A}\,A+\overline{A}\,B+\overline{B}\,A+\overline{B}\,B)$$

$$Y=A\overline{B}+C\,\overline{A}+C\,\overline{B}+(\overline{B}C+\overline{C}\,B)(\overline{A}\,B+\overline{B}\,A)$$

$$Y=A\overline{B}+C\,\overline{A}+C\,\overline{B}+\overline{B}C\,\overline{A}\,B+\overline{B}C\,\overline{B}\,A+\overline{C}\,B\,\overline{A}\,B+\overline{C}\,B\,\overline{B}\,A$$

$$Y=A\overline{B}+C\,\overline{A}+C\,\overline{B}+\overline{A}\,B\,\overline{B}C+A\overline{B}C+\overline{A}\,B\overline{C}+A\,B\,\overline{B}\,\overline{C}$$

$$Y=A\overline{B}+C\,\overline{A}+C\,\overline{B}+A\overline{B}C+\overline{A}\,B\overline{C}$$

$$Y=A\overline{B}+C\,\overline{A}+C\,\overline{B}+\overline{A}\,B\overline{C}$$

$$Y=A\overline{B}+\overline{A}(C+B\overline{C})+C\,\overline{B}$$

Assorbimento 2:

T8" $X+\overline{X}\,Y=X+Y$

T8"' $X\,(\overline{X}+Y)=X\,Y$

$$Y=A\overline{B}+\overline{A}(C+B)+C\,\overline{B}$$

$$Y=A\overline{B}+\overline{A}C+\overline{A}\,B+C\,\overline{B}$$

$$Y=A\overline{B}+\overline{A}C+\overline{B}C+\overline{A}\,B$$

T11', consenso :

$$XY+\overline{X}\,Z+YX=XY+\overline{X}\,Z$$

$$Y=A\overline{B}+\overline{A}C+\overline{A}\,B$$

$$Y=A\overline{B}+\overline{A}\,B+\overline{A}C$$

$$Y=A\oplus B+\overline{A}C$$

Forma minima